# On Gas Separation in Ranque–Hilsch Vortex Tubes

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Partial separation of a gas mixture in the vortex tube has been reported by a number of investigators [1–7]; however, consistent results especially with tubes working at atmospheric pressure and above have been difficult to obtain [5]. In particular it has been a puzzling feature of the studies that the heavy component has been found to be concentrated sometimes in the hot stream (the "normal" effect) and sometimes in the cold stream (thte "inverse" effect). From extensive measurements [8] of the separation obtained with the type of tube shown in Fig. 1 (see typical results in Figs. 4 and 5), and an experimental study [9] of the tangential velocity distribution in the tubes, the following description of the process by which the measured net separation may be found, has been developed (see also [10]). This description is based on the conclusion reached in [8] that centrifugation is the primary cause of the separation, but that secondary flows, notably in the axial directions, have a profound influence on the actual effect measured between the outlets.

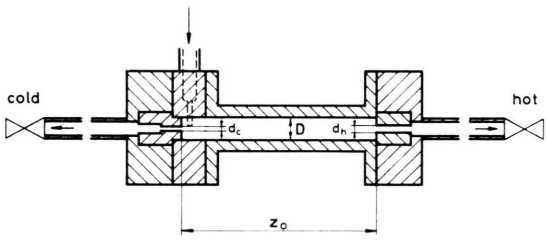


Fig. 1. Vortex tube, with valves and exchangeable orifices at both ends. "Cold" end (by convention) adjacent to tangential inlet jet.

A model of the secondary flow pattern has been developed [10] based on the experimental finding (see f. ex. [11] and [12]), that the boundary layers at the end walls of the tube carry a substantial proportion of the total radial influx, a boundary layer flow which to a large extent is re-ejected into the tube as turbulent axial flow at radii equal to or somewhat greater than the exit radius (see Fig. 2). This axial flow retains its identity along the tube so that a kind of double counter-

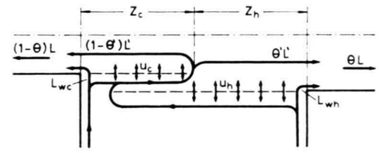


Fig. 2. Schematic representation of separative exchange region in tube with $d_h > d_c$ when both valves are open ($\Theta = \Theta_0$). Heavy lines show relevant flows; tangential velocity component not included. Cross section of tube limited (top) by centre axis and (left and right) by end walls with exit ducts; peripheral wall with inlet nozzle not shown. $z_h + z_c = z_0$ (Fig. 1).

current system is created as indicated on the figure (where the cold end, with the inlet jet, by the lettering is placed towards the left, though in actual fact the jet may be positioned anywhere along the tube without change in the pattern shown). The diffusion caused by the centrifugal field, between the axial streams will lead to concentration changes in the outgoing streams. Taking into account that the total mass flow through the tube is very large so that a back-diffusion caused by the concentration gradients may be neglected, the rate of separative diffusion (measured per cm tube length) across a cylinder surface such as $r = r_c$ or $r = r_h$, is

$$u = 2\pi \frac{D_f P}{RT} \frac{\Delta M}{RT} v^2 N(1-N) \text{ Mole sec}^{-1} \text{ cm}^{-1} \quad (>0) \tag{1}$$

where $D_f$ is the coefficient of diffusion, $\Delta M$ is the molecular weight difference, $v$ is the tangential velocity at the surface and $N$ the mole fraction of one of the components of the gas mixture; quantities that all are experimentally available.

According to the model, and in agreement with experiment [8], the outer region of the tube (with an inward-moving flow not shown on Fig. 2) does not contribute to the detected net separation because of the mixing taking place in the axial flows nearer the axis.

A prediction of the separation in the case shown on Fig. 2 cannot be made a priori because $z_h/z_c$ is not known, but it should be noted that the two countercurrent "columns" ($u_c$ and $u_h$) at work in the tube oppose one another so that even the "sign" of the net separation effect is indeterminate, i. e. either one of the two streams may become the heavier (below, $z_h/z_c$ is treated as an adjustable parameter).

The next step in the description takes into account that a change in $\Theta$ (the flow fraction through the hot exit) is brought about by partly closing one or the other valve (Fig. 1), i. e. by a pressure increase at the end of the exit duct being closed. Due to the radial

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pressure gradient in the tube itself, this pressure increase in the duct will first affect the outflow along the centre axis, conceivably stop it altogether and reverse it without reducing the outflow at the periphery of the exit duct appreciably. This must change the position of the stagnation point along the axis and a simple pattern that may result under these conditions is indicated in Fig. 3 for a "symmetrical" tube, i. e. one with equal orifices at the two ends. In Fig. 3 a the cold exit, in Fig. 3 b the hot exit is partly closed (it follows that Fig. 2 applies to the special case with both valves open). It is seen that in Fig. 3 a or 3 b a third separating counter-current region has been established, a region gaining in width and therefore importance the more the valve is closed.

From mass balance considerations, and with simple assumptions about the magnitude of the reversed flow $L_h^*$ or $L_c^*$ (Fig. 3) and the radius $r^*$ (limiting the centre region with reversed flow) as functions of $\Theta - \Theta_0$ (where $\Theta_0$ is $\Theta$ when both exits are unrestricted), the following equation is obtained for the net separation effect (see [10] for further details)

$$\mathrm{d}N \equiv N_h - N_c = \frac{1}{\Theta(1-\Theta)\,L} \cdot [\Theta' L' (N_h' - N_0') - \Theta' L_{wc}\,\mathrm{d}N_{wc} + (1-\Theta')\,L_{wh}\,\mathrm{d}N_{wh}]. \quad (2)$$

Here $L$ is the total flow through the tube, $L' = L - L_{wc} - L_{wh}$ (Fig. 2 or 3), $\Theta' = (\Theta L - L_{wh})/L'$, $N$ is mole fraction of the heavy component, $\mathrm{d}N_{wc}$ and $\mathrm{d}N_{wh}$ are (relatively small) changes in mole fraction, possibly of non-centrifugal origin, experienced by the end wall boundary layer flows [10], and $\Theta' L' (N_h' - N_0')$, the net transfer of heavy component from cold to hot stream (excluding the end wall flows $L_{wc}$ and $L_{wh}$), is determined either, for $\Theta' > \Theta_0'$ (Fig. 3 a) by

$$\Theta' L' (N_h' - N_0') = -(1-\Theta')\left(u_c z_c + u_h z_h + \frac{1}{(1-\Theta_0')}(u^* - u_c)\, z_c\right) \quad (3\,a)$$

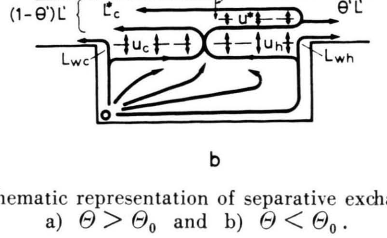


Fig. 3. Schematic representation of separative exchange when a) $\Theta > \Theta_0$ and b) $\Theta < \Theta_0$.

or, for $\Theta' < \Theta_0'$ (Fig. 3 b) by

$$\Theta' L' (N_h' - N_0') = \Theta'\left(u_c z_c + u_h z_h + \frac{1}{\Theta_0'}(u^* - u_h)\, z_h\right). \quad (3\,b)$$

Here $\Theta_0'$ is $\Theta'$ when both valves are fully open, and the meaning of the remaining letters is as shown in Figs. 3 a and b.

The theory has been tested by comparing calculated curves, based on flow dynamic data, with experimental results (see [10] and Figs. 4 and 5). Quite close agreement is in general obtained, and an interpretation of the experimental results along the following lines may therefore be attempted.

The appearance of both positive ($N_h > N_c$) and negative ($N_h < N_c$) effects is a consequence of the symmetry properties of the axial flow the shift from plus to minus with increasing $\Theta$ being determined primarily by the flow reversal on the centre axis at $\Theta = \Theta_0$. The existence of extremas on the curves with a return to small effects at low and high $\Theta$ is to be

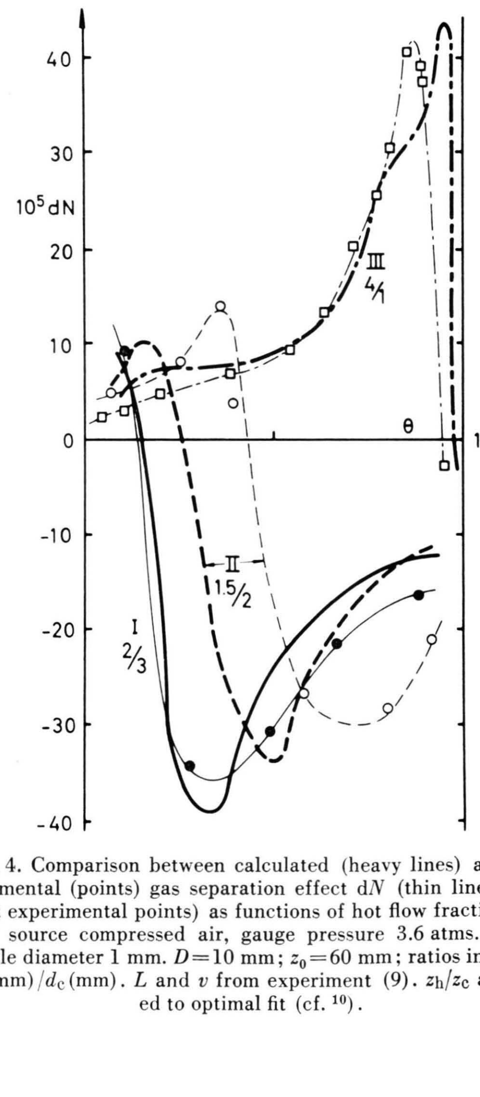


Fig. 4. Comparison between calculated (heavy lines) and experimental (points) gas separation effect $\mathrm{d}N$ (thin lines connect experimental points) as functions of hot flow fraction, $\Theta$. Gas source compressed air, gauge pressure 3.6 atms.; inlet nozzle diameter 1 mm. $D$=10 mm; $z_0$=60 mm; ratios indicate $d_h$(mm)/$d_c$(mm). $L$ and $v$ from experiment (9). $z_h/z_c$ adjusted to optimal fit (cf. [10]).

ascribed mainly to the effect of the admixed end wall flows.

The point of inversion (neg./pos.) varies rapidly with $d_h/d_c$ (Fig. 4) because $\Theta_0$, as shown by experiment [8], increases rapidly with $d_h/d_c$.

The numerical values of the maximum and/or minimum tend to increase with decreasing orifice diameter (Fig. 5); the model relates this change to the fact that higher tangential velocities are obtained in the separating region the smaller the orifices are (see [9]).

In [10] it is, furthermore, shown that the theory accounts in a satisfactory manner for the dependence of the separation on tube length, i. e. the experimental finding that maximum separation is obtained with relatively short tubes [9] (see also [6]).

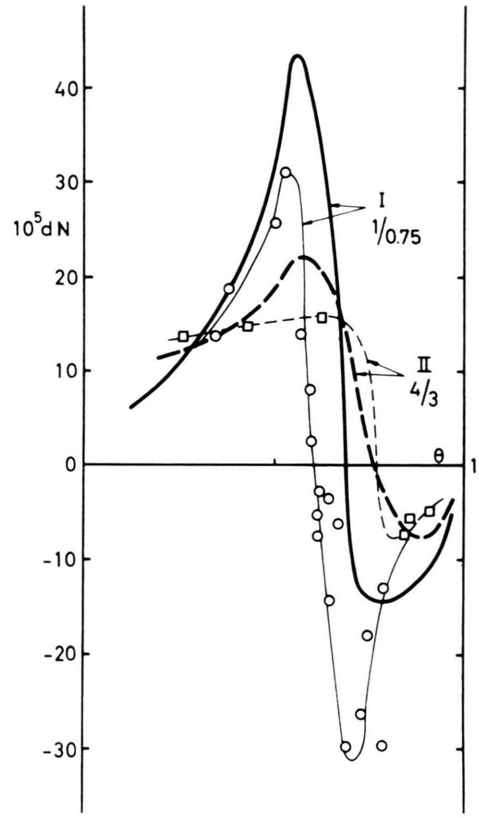


Fig. 5. Comparison between calculated and experimental gas separation effect $d_h/d_c = 1.33$; otherwise as Fig. 4.

# Isotope Transport along a Temperature Gradient in Li Metal

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It has been found that, when a temperature gradient is maintained along a Li metal rod, the light isotope becomes enriched in the hot portion. This is contrary to expectation, as it is known that thermotransport motion of Li atoms relative to the lattice is directed towards the cold portion. It is inferred that isotope thermotransport and bulk thermotransport are governed by two different mechanism. The isotope factor [defined as $a = (\Delta v/v)/(\Delta M/M)$, where $\Delta$ stands for isotope differences, $v$ for transport velocities, $M$ for mass numbers] is of the order $a \approx +0.35$.

A year ago it has been reported [1] that if alternating current is passed along a Li metal rod, the ends of which are force-cooled, the temperature gradient causes a considerable flow of Li atoms (thermotransport) away from the hot zone. It was to be expected that, as in all diffusive motion, this flow should be accompanied by an isotope effect. This year the isotopic compositions of three Li rods have been investigated after about one month's thermotransport anneal. Before sectioning for mass analysis, each sample presented a typical "hour-glass" appearance. Because of atom transport away from the hottest point, the "waist" diameter had diminished by some 10%. At this portion of each specimen, the change in isotopic composition was found to be greatest, amounting to an enrichment of the *light* isotope by about 2%. The heavy isotope was found to be enriched where the specimens had become thicker.

A convenient way of expressing isotope transport results is by means of the "isotope factor", defined as

$$a = \frac{\Delta v/v}{\Delta M/M}, \quad (1)$$

where $v$ is the atom transport velocity wrt. the lattice, $\Delta v$ the differences in migration velocity of two isotopes, whose mass difference is $\Delta M$.

This factor can be derived from experimental data according to the following arguments:

Let the concentration of isotope $i$ ($i = 6$ or 7) at a length coordinate $x$ of the metal rod be $c_i$ and the time independent isotope velocity $v_i$. Let $x + \delta x$ correspond to $c_i + \delta c_i$ and to $v_i + \delta v_i$. The primary results of the

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